Calculus 1 Workbook

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krista king
MATH

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■ 1. A truck is 40 miles north of an intersection, traveling toward the intersection at 35 mph. At the same time, another car is 30 miles west of the intersection, traveling away from the intersection at 45 mph. Is the distance between the vehicles increasing or decreasing at that moment? At what rate?

■ 2. Water is flowing out of a cone-shaped tank at a rate of 6 cubic inches per second. If the cone has a height of 5 inches and a base radius of 4 inches, how fast is the water level falling when the water is 3 inches deep?

■ 3. A ladder 25 feet long leans against a vertical wall of a building. If the bottom of the ladder is pulled away horizontally from the building at 3 feet per second, how fast is the angle formed by the ladder and the horizontal ground decreasing when the bottom of the ladder is 7 feet from the base of the wall?

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APPLIED OPTIMIZATION

■ 1. A boater finds herself 2 miles from the nearest point to a straight shoreline, which is 10 miles down the shore from where she parked her car. She plans to row to shore and then walk to her car. If she can walk 4 miles per hour but only row 3 miles per hour, toward what point on the shore should she row in order to reach her car in in the least amount of time?

■ 2. Mr. Quizna wants to build in a completely fenced-in rectangular garden. The fence will be built so that one side is adjacent to his neighbor's property. The neighbor agrees to pay for half of that part of the fence because it borders his property. The garden will contain 432 square meters. What dimensions should Mr. Quizna select for his garden in order to minimize his cost?

■ 3. A company is designing shipping crates and wants the volume of each crate to be 6 cubic feet, and each crate's base to be a square between 1.5 feet and 2.0 feet per side. The material for the bottom of the crate costs $\$5$ per square foot, the sides $\$3$ per square foot, and the top $\$1$ per square foot. What dimensions will minimize the cost of the shipping crates?

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MEAN VALUE THEOREM FOR DERIVATIVES

■ 1. Find the value(s) of c that satisfy the Mean Value Theorem for the function in the interval $[1,5]$.

$$f(x) = x^3 - 9x^2 + 24x - 18$$

■ 2. Find the value(s) of c that satisfy the Mean Value Theorem for the function in the interval $[1,4]$.

$$g(x) = \frac{x^2 - 9}{3x}$$

■ 3. Find the value(s) of c that satisfy the Mean Value Theorem for the function in the interval $[0,5]$.

$$h(x) = -\sqrt{25 - 5x}$$

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ROLLE'S THEOREM

■ 1. Use Rolle's Theorem to show that the function has a horizontal tangent line in the interval $[-1,2]$. Find the value(s) of c in the interval that satisfy Rolle's Theorem.

$$f(x) = x^3 - 2x^2 - x - 3$$

■ 2. Use Rolle's Theorem to show that the function has a horizontal tangent line in the interval $[-3,5]$. Find the value(s) of c in the interval that satisfy Rolle's Theorem.

$$g(x) = \frac{x^2 - 2x - 15}{6 - x}$$

■ 3. Use Rolle's Theorem to show that the function has a horizontal tangent line in the interval $[-\pi/2,\pi/2]$. Find the value(s) of c in the interval that satisfy Rolle's Theorem.

$$h(x) = \sin(2x)$$

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NEWTON'S METHOD

■ 1. Use four iterations of Newton's method to approximate the root of $g(x) = x^3 - 12$ in the interval $[1,2]$. Give the answer to the nearest three decimal places.

■ 2. Use four iterations of Newton's method to approximate the root of $f(x) = x^4 - 15$ in the interval $[-2, -1]$. Give the answer to the nearest four decimal places.

■ 3. Use four iterations of Newton's method to approximate the root of $h(x) = 3e^{x-4} - 8$ in the interval $[4,6]$. Give the answer to the nearest four decimal places.

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L'HOSPITAL'S RULE

■ 1. Use L'Hospital's Rule to evaluate the limit.

$$\lim_{x\to 0} \frac{2\sqrt{x+4} - 4 - \frac{1}{2}x}{x^2}$$

■ 2. Use L'Hospital's Rule to evaluate the limit.

$$\lim_{x\to \frac{\pi}{2}} \frac{\sec x}{3 + \tan x}$$

■ 3. Use L'Hospital's Rule to evaluate the limit.

$$\lim_{x\to \infty} \frac{\ln x}{4\sqrt{x}}$$

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POSITION FUNCTION, SPEED, VELOCITY, ACCELERATION

■ 1. Find the velocity $v(t)$, speed, and acceleration $a(t)$ at $t = 2$ of the position function.

$$s(t) = -\frac{t^3}{3} + t^2 + 3t - 1$$

■ 2. Find the velocity $v(t)$, speed, and acceleration $a(t)$ at $t = 1$ of the position function.

$$s(t) = \frac{t^2 - 3}{t^3}$$

■ 3. Find the velocity $v(t)$, speed, and acceleration $a(t)$ at $t = 4$ of the position function.

$$s(t) = \frac{t^2}{2t + 4}$$

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VERTICAL MOTION, BALL THROWN UP FROM THE GROUND

■ 1. A ball is thrown straight upward from the ground with an initial velocity of $v_0 = 86$ ft/sec. Assuming constant gravity, find the maximum height, in feet, that the ball attains, the time, in seconds, that it's in the air, as well as the ball's velocity, in ft/sec, when it hits the ground.

■ 2. A ball is thrown straight upward from the top of a building, which is 56 feet above the ground, with an initial velocity of $v_0 = 48$ ft/sec. Assuming constant gravity, find the maximum height, in feet, that the ball attains, the time, in seconds, that it's in the air, as well as the ball's velocity, in ft/sec, when it hits the ground.

■ 3. A ball is thrown straight upward from a bridge, which is 24 meters above the water, with an initial velocity of $v_0 = 20$ m/sec. Assuming constant gravity, find the maximum height, in meters, that the ball attains, the time, in seconds, that it's in the air, as well as the ball's velocity, in m/sec, when it hits the water below.

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VERTICAL MOTION, COIN DROPPED FROM THE ROOF

■ 1. A rock is dropped from the top of an 800 foot tall cliff, with an initial velocity of $v_0 = 0$ ft/sec. Assuming constant gravity, when does the rock hit the ground, and what is its velocity when it hits the ground?

■ 2. A rock is tossed from the top of a 300 foot tall cliff, with an initial velocity of $v_0 = 15$ ft/sec. Assuming constant gravity, when does the rock hit the ground, and what is its velocity when it hits the ground?

■ 3. A coin is tossed downward from the top of a 36 meter tall building, with an initial velocity of $v_0 = 6$ m/sec. Assuming constant gravity, when does the rock hit the ground, and what is its velocity when it hits the ground?

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MARGINAL COST, REVENUE, AND PROFIT

■ 1. A company manufactures and sells basketballs for $\$9.50$ each. The company has a fixed cost of $\$395$ per week and a variable cost of $\$2.75$ per basketball. The company can make up to 300 basketballs per week. Find the marginal cost, marginal revenue, and marginal profit, if the company makes 150 basketballs.

■ 2. A company manufactures and sells high end folding tables for $\$250$ each. The company has a fixed cost of $\$3{,}000$ per week and variable costs of $85x + 150\sqrt{x}$, where x is the number of tables manufactured. The company can make up to 200 tables per week. Find the marginal cost, marginal revenue, and marginal profit, if the company makes 64 tables.

■ 3. A company manufactures and sells electric food mixers for $\$150$ each. The company has a fixed cost of $\$7{,}800$ per week and variable costs of $24x + 0.04x^2$, where x is the number of mixers manufactured. The company can make up to 200 mixers per week. Find the marginal cost, marginal revenue, and marginal profit, if the company makes 75 mixers.

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HALF LIFE

- 1. Find the half-life of Tritium if its decay constant is 0.0562.

- 2. Find the half-life of Cobalt-60 if its decay constant is 0.1315.

- 3. Find the half-life of Berkelium-97 if its decay constant is 0.000503.

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NEWTON'S LAW OF COOLING

■ 1. A cup of coffee is 195° F when it's brewed. Room temperature is 74° F. If the coffee is 180° F after 5 minutes, to the nearest degree, how hot is the coffee after 25 minutes?

■ 2. A boiled egg that's 99° C is placed in a pan of water that's 24° C. If the egg is 62° C after 5 minutes, how much longer, to the nearest minute, will it take the egg to reach 32° C.

■ 3. Suppose a cup of soup cooled from 200° F to 161° F in 10 minutes in a room whose temperature is 68° F. How much longer will it take for the soup to cool to 105° F?

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COMPOUNDING INTEREST

■ 1. Suppose you borrow $\$15{,}000$ with a single payment loan, payable in 2 years, with interest growing exponentially at $1.82\,\%$ per month, compounded continuously. How much will it cost to pay off the loan after 2 years?

■ 2. Your parents deposit $\$5{,}000$ into a college savings account, with interest growing exponentially at $0.875\,\%$ per quarter, compounded continuously. How much will be in the account after 18 years?

■ 3. Suppose you win $\$50{,}000$ in a contest and you decide to save it for your retirement. You deposit it into an annuity account that pays $2.4\,\%$ semi-annually, compounded continuously. How much will the account contain after 25 years, when you plan to retire?

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SALES DECLINE

■ 1. Suppose a pizza company stops a special sale for their three-topping pizza. They will resume the sale if sales drop to $70\,\%$ of the current sales level. If sales decline to $90\,\%$ during the first week, when should the company expect to start the special sale again?

■ 2. Suppose a donut store experiments with raising the price of a dozen donuts to see if sales are affected. They'll resume the sale if sales drop to $80\,\%$ of the current sales level. If sales decline to $90\,\%$ after two weeks, when should the store change back to the original price?

■ 3. Suppose a flower shop decides to stop ordering roses in the winter time to see if sales are affected. They will resume the sale if sales drop to $90\,\%$ of the current sales level. If sales decline to $96\,\%$ after three weeks, when should the shop begin ordering roses again?

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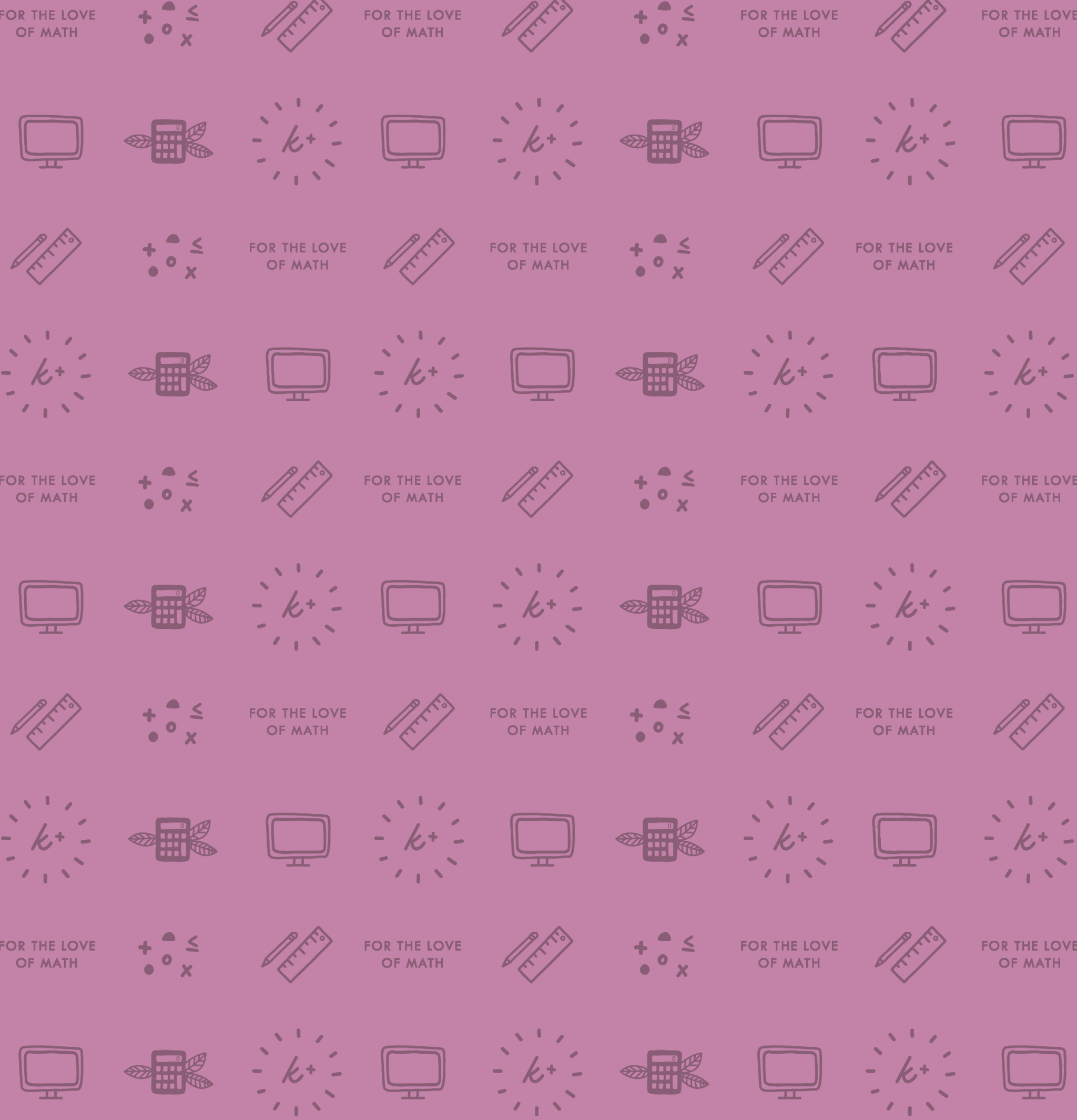

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